

Class Notes

Applications of the Normal Distribution (Section 7.2)

What is the probability
that an adult male is at
least six feet tall?

In the last section, we explored this curve called the normal density function. Here, we see many examples of it in use. We may have to use the complement rule or other reasoning to answer certain questions.

Now, the normal curve always has a nice bell-like shape. However, its exact dimensions are determined by the mean and standard deviation of the distribution. What that means is that we would need some fancy math to find the areas under these curves.

Instead of doing that, we will look up these areas on tables (or use technology). We cannot have a table for each and every possible mean and standard deviation. So we reduce our normal distribution to the standard normal distribution, for which we do have a table. This is the process of finding a z-score as we have done before.

Definition: Standardizing a Normal Random Variable: Suppose that the random variable X is normally distributed with mean μ and standard deviation σ . Then the random variable

$Z = \frac{X - \mu}{\sigma}$ is normally distributed with a mean $\mu = 0$ and standard deviation $\sigma = 1$. The random variable Z is said to have the standard normal distribution.

Table V in the book (page A-11) gives us the area under this curve for various z-scores. Also, it exists on the fold-out flap at the back of the book.

Optional Worksheet: Normal distribution:

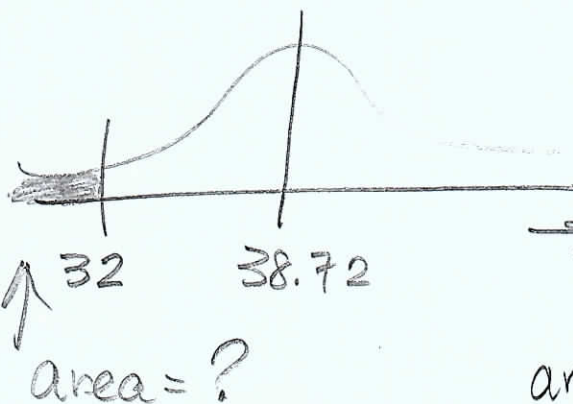
This optional worksheet, started in section 7.1, will guide us through finding areas under the curve using the table. It also has a few practice problems.

expl 1a: A pediatrician obtains the heights of her three-year-old patients. The heights are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. We will find the proportion of her patients that are less than 32 inches tall.

$X = \text{height of patients.}$

$\sigma = 3.17$
 $\mu = 38.72 \text{ in.}$

Draw a normal curve with the mean of 38.72 and the observation 32 marked. Shade the area we are interested in.



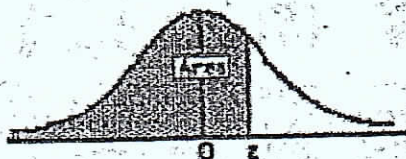
Step 1: Draw a normal curve so we see what area we want.

→ from (#1b)

The proportion of patients that are less than 32 in tall is 0.0170 (or 1.7%).

Here is a picture of part of the Table V. We will use it to find the area we are after. But we need to convert our observation into a z-score first.

Area Under a Normal Curve to the Left of z, Where $z = \frac{x - \mu}{\sigma}$



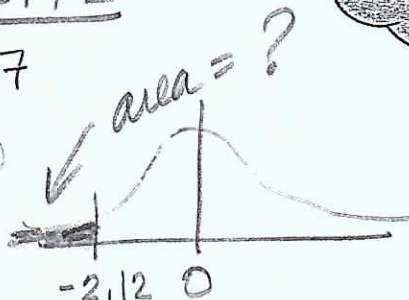
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0052	.0050	.0049	.0047	.0046	.0044	.0043	.0041	.0040	.0038
-2.4	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.3	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0065	.0064
-2.2	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.1	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.0	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-1.9	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.7	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0446	.0438	.0429	.0420	.0411	.0401	.0392	.0384	.0375	.0367

expl 1b: Use the formula $z = \frac{x - \mu}{\sigma}$ to find the z-score for this observation of 32. We take our z-score, rounded to two decimal places, and look it up in the table. We find the ones and tenths digits along the first column marked "z". Then follow along that row to find the hundredths place (labeled at top) of our z-score. The number in the correct row and column of the table is the area we are after. Circle it above in the table. Write it down here in sentence form.

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{32 - 38.72}{3.17}$$

$$z \approx -2.12$$



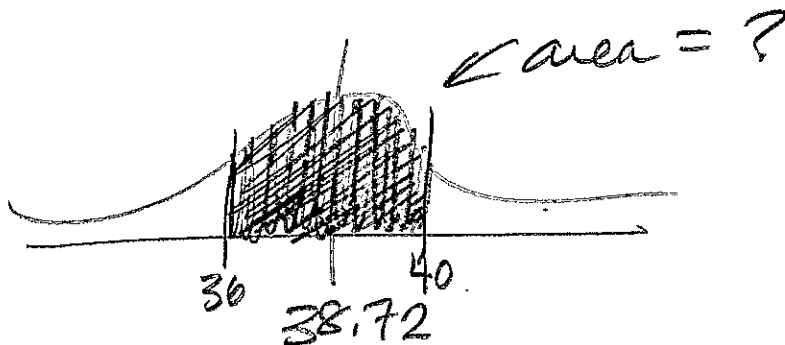
Step 2: Convert this "observation" of 32 into a z-score.

Step 3: Look the z-score up in the table.

area ≈ 0.0170

expl 2: Consider the same pediatrician and her patients. Recall the heights of her three-year-old patients are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. Find the probability that a randomly selected patient is between 36 and 40 inches tall. Follow the steps below.

a.) Draw a normal curve with the given mean and observations marked. Shade the area we need.



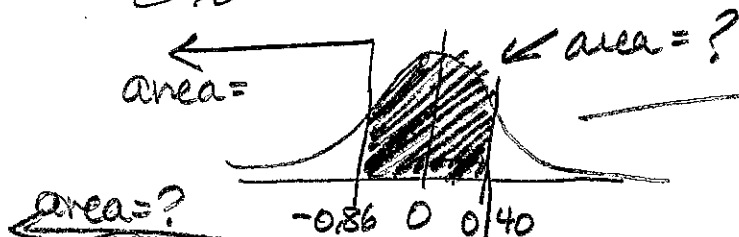
b.) Find the z-scores for both observations of 36 and 40.

$$z = \frac{36 - 38.72}{3.17}$$

$$z = \frac{40 - 38.72}{3.17}$$

$$z \approx -0.86$$

$$z \approx 0.40$$



area to left of 0.40
- area to left of -0.86

$$= 0.6554 - 0.1949$$

$$= 0.4605$$

c.) How do you find the area we need by only using the table? A complete table is provided at the end of these notes. Write a sentence with your final answer.

46.05% of these patients are between 36 and 40 inches tall.

(7.2 pg 4)

Calculator and StatCrunch Instructions:

normal pdf is only used for graphing curves on x-y plane (don't need)

TI calculators Instructions:

Press 2nd VARS to access the DISTR menu. Select normalcdf and select it to put it on the home screen.

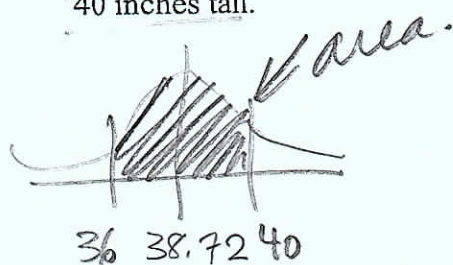
You'll need to enter the values for the lowerbound, upperbound, μ , and σ , following each with a comma (directly above the 7 button). End the expression with a parenthesis and press **ENTER**. Newer calculators may have a slightly different interface.

StatCrunch Instructions:

Select **Stat > Calculators > Normal**. Enter the mean and standard deviation. It also has a spot to form the probability you want to find. Play around with the tabs **Standard** and **Between** to see what they do. You can change the inequality symbols as needed.

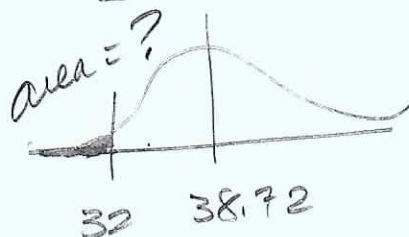
This is a repeat of examples 1 and 2. We will do them this time with technology.

expl 3a (example 2, again): Consider the same pediatrician and her patients. Recall the heights of her three-year-old patients are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. Find the probability that a randomly selected patient is between 36 and 40 inches tall.



$$\text{normalcdf}(36, 40, 38.72, 3.17)$$
$$\approx 0.4614$$

expl 3b (example 1, again): Consider the same pediatrician and her patients. Recall the heights of her three-year-old patients are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. We will find the proportion of her patients that are less than 32 inches tall.



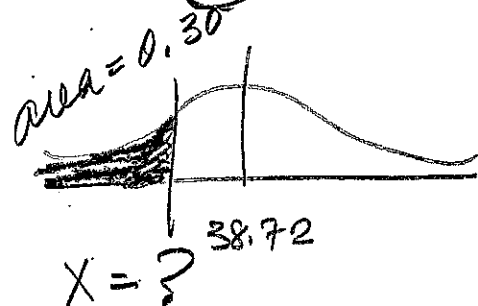
Consider your graph. What is the **lowerbound**?

$$\text{normalcdf}(0, 32, 38.72, 3.17)$$
$$\approx 0.0170$$

$X = ?$

expl 4: Consider the same pediatrician and her patients. Recall the heights of her three-year-old patients are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. Find the height of a patient at the 30th percentile. Follow these steps.

a.) Draw a normal curve and shade the area in which we are interested.



What does 30th percentile mean?

Here, we are given an area and are asked to find the observation that corresponds to it.

b.) Look in the body of the Table V to find an area as close to .3000 as we can get. Write down the z-score to which it corresponds. (If there are two equally close z-scores, we average them.)

$$z = -0.52$$

c.) Use the formula $z = \frac{x - \mu}{\sigma}$ to convert this z-score into a value of x . Answer in sentence form.

$$-0.52 = \frac{x - 38.72}{3.17}$$

$$x \approx 37$$

A child who is 37 inches tall is at the 30th percentile. (Meaning, 30% of all ^{the} 3-yr-old patients are less than or equal to 37 inches tall.)

Calculator and StatCrunch Instructions:

TI calculators Instructions:

Press 2nd VARS to access the DISTR menu. Select invNorm and select it to put it on the home screen.

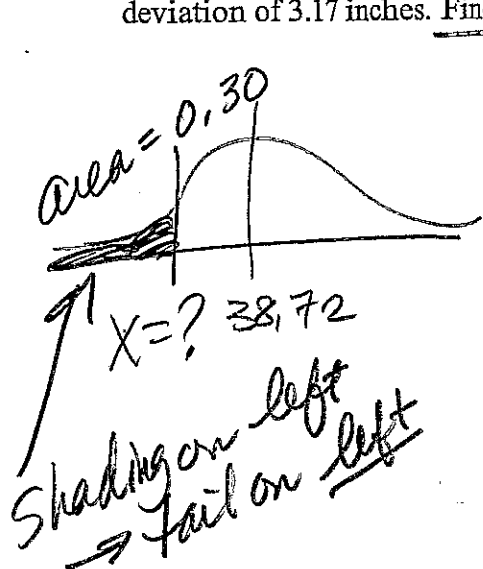
You'll need to enter the values for the area left (in decimal form), μ , and σ , following each with a comma (directly above the 7 button). End the expression with a parenthesis and press ENTER. Newer calculators may have a slightly different interface.

StatCrunch Instructions:

Select Stat > Calculators > Normal. Enter the mean and standard deviation. Enter the area we want (in decimal form as .3) in the space after the equal sign. Play around with the tabs Standard and Between to see what they do. You can change the inequality symbols as needed. Press Compute! and it should give you the correct x -value.

This is a repeat of example 4. We will do it this time with technology. We do not need to go through the z-score step because we will use the original distribution's mean and standard deviation.

expl 5 (example 4 again): Consider the same pediatrician and her patients. Recall the heights of her three-year-old patients are normally distributed with a mean of 38.72 inches and a standard deviation of 3.17 inches. Find the height of a patient at the 30th percentile.



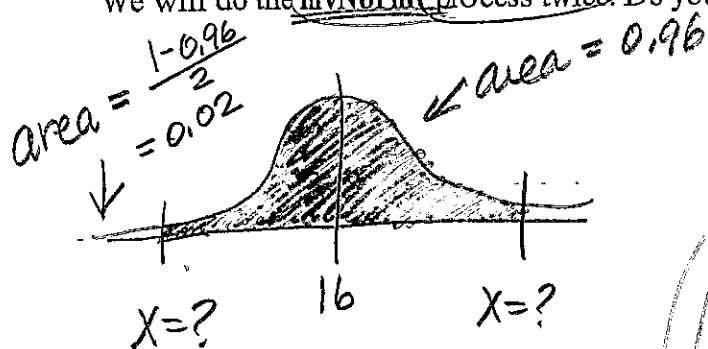
$$\text{invNorm}(0.30, 38.72, 3.17)$$

$$X \approx 37 \text{ inches}$$

If you have a newer calculator, play around with the settings for InvNorm

expl 6: The mean time for service at a certain oil change shop is 16 minutes with a standard deviation of 2.3 minutes. Use technology to answer the following question. Between which two times will the middle 96% of the visits fall? Draw a normal curve with the appropriate area shaded. Round to one decimal place and write a sentence.

We will do the invNorm (process twice). Do you see why?



Split $100\% - 96\%$ or 4% between the tails of the curve.

the middle
So, 96% of the visits will fall between 11.3 and 20.7 minutes.

$$\text{invNorm}(0.02, 16, 2.3)$$

$$X \approx 11.3 \text{ minutes}$$

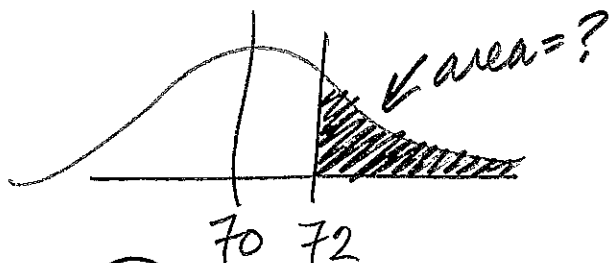
$$\text{area} = 0.98$$

$$\text{invNorm}(0.98, 16, 2.3)$$

$$X \approx 20.7 \text{ minutes}$$

expl 7: Men's heights are normally distributed with a mean of 70 inches and a standard deviation of 4 inches. Find the probability that a randomly selected man is more than six feet tall. Follow these steps.

a.) Draw a normal curve with the mean and 72 inches (six feet) marked. Shade the area we need.



Be careful. How is this different than what we have done?

b.) Use technology to find the area we want. Within the normalcdf command (described on page 4), enter E99 (this is 1 followed by 99 zeros) or some other ridiculously high number as the upper bound. (Use EE, the second function of the comma.)

$$\text{normalcdf}(72, 128, 70, 4) \\ \approx 0.3085$$

Alternatively, we could use the book's table. How would that work?

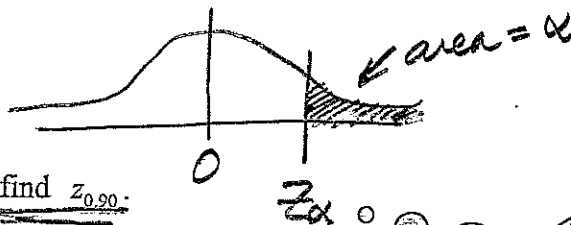
→ So, 30.85% of guys are more than 6 feet tall.

Worksheet: Normal distribution practice 3:

This worksheet will practice many normal distribution applications.

Definition: The notation z_α (pronounced "z sub alpha") is the z-score such that the area under the standard normal curve to the right of z_α is α .

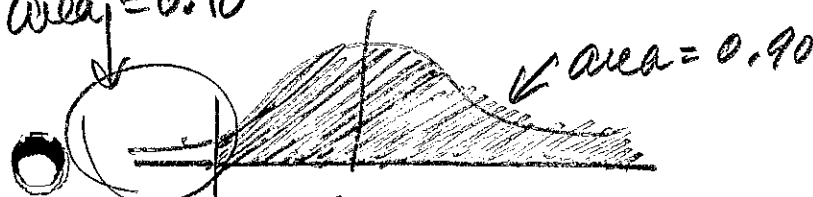
Draw yourself a picture of this situation.



z_α

expl 8: Use Table V to find $z_{0.90}$.

area = 0.10



Think about what the 0.90 means. What should we look up on the table?

$z_{0.90}$

From table,

$$z_{0.90} \approx -1.28$$

$$1 - 0.90 \\ = 0.10$$

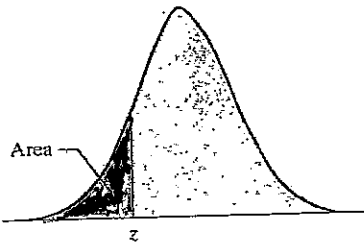


Table V

Standard Normal Distribution										
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

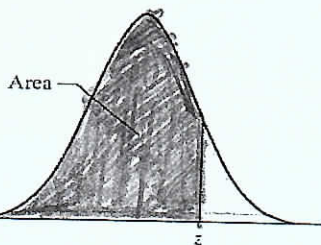


Table V (continued)

Standard Normal Distribution										
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

(b) Confidence Interval Critical Values, $z_{\alpha/2}$

Level of Confidence	Critical Value, $z_{\alpha/2}$
0.90 or 90%	1.645
0.95 or 95%	1.96
0.98 or 98%	2.33
0.99 or 99%	2.575

(c) Hypothesis Testing Critical Values

Level of Significance, α	Left-Tailed	Right-Tailed	Two-Tailed
0.10	-1.28	1.28	± 1.645
0.05	-1.645	1.645	± 1.96
0.01	-2.33	2.33	± 2.575

